

EQUIVALENT CIRCUITS FOR DISCONTINUITIES IN BALANCED STRIP TRANSMISSION LINE

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Abstract

Theoretical formulas are derived for the equivalent circuit parameters of a variety of discontinuities in balanced strip transmission line. These formulas are simple in form and are obtained by employing a small aperture procedure or a Babinet equivalence procedure in conjunction with an approximate model of the line. The results for a number of discontinuities are presented and comparison is made with the available measured data.

1. Introduction

Approximate procedures are employed in this paper for the determination of equivalent circuits for a variety of discontinuity structures in balanced strip transmission line. Before discussing these structures or the theoretical procedures, it is desirable to mention certain general qualitative considerations. Many of these general remarks are well-known to those working with strip lines.

1. Discontinuities in balanced strip transmission line will possess purely reactive equivalent networks if a) the discontinuity is balanced, b) the outer plate spacing is less than $\lambda/2$, and, of course, c) the discontinuity structure contains no dissipative elements. If the discontinuity structure is unbalanced (unsymmetrical with respect to the upper and lower portions of the line), the discontinuity will excite the dominant mode in radial transmission line (the lowest E-type (so-called pillbox) mode, similar to the TEM mode in parallel plane line), and radiation will occur producing resistive contributions to the equivalent circuit.

A little of this unbalance can be tolerated and the radiation prevented by boxing in a small region surrounding the discontinuity so that the unwanted excited mode becomes the dominant mode in rectangular waveguide, and by choosing the dimensions such that the mode is below cut-off. Even a balanced discontinuity will excite a variety of higher radial line modes, the lowest of these being similar to the E_1 (or TM_1) mode in parallel plate line. None of these modes will propagate, however, if the plate spacing is less than $\lambda/2$. Therefore, although the propagation of the dominant mode does not impose any condition on the plate spacing, a practical system must have a plate separation less than $\lambda/2$.

2. Most of the so-called transverse discontinuity structures (those which occupy a portion of the cross-section plane) in common use in coaxial line or rectangular guide do not have a counterpart in strip line because of constructional difficulties. As a result, it is difficult in strip line to obtain a shunt capacitive discontinuity; a shunt inductance can be produced by use of a vertical wire or post. On the other hand, it is easy in strip line to obtain either series inductances or capacitances.

3. It is possible also to produce in a simple fashion a series of resonant discontinuity. An example of how this may be obtained is the following. A narrow gap extending completely across the inner conductor is known to be a series capacity (see Fig. 1(a)). On the other hand, the rectangular slot of Fig. 1(b) (which may be photo-etched without any difficulty) is a serious inductance. The proper choice of length l of the slot will produce resonance. It is of interest to note that this length for resonance must of necessity be considerably less than $\lambda/2$.

4. The dominant mode in balanced strip line is a TEM mode so that the field distribution can be determined rigorously by conformal mapping procedures. One cannot apply these methods to the higher modes, however, so that it is quite difficult to obtain the field distributions of even the first few higher modes, although their cut-off wavelengths may be fairly readily computed. For the rigorous solution of discontinuity problems one must have available the approximate Green's function for the region. Since the geometry of strip line is not separable, as it is for most waveguide shapes, the rigorous determination of the Green's function is a major task in itself. For this reason it becomes advisable to resort to approximation schemes in the determination of the equivalent circuit parameters of discontinuity structures in strip line. All of the efforts described herein have been along these latter lines.

A qualitative description of the theoretical procedures employed in obtaining the equivalent circuit parameters of a number of discontinuity structures in strip line is given in Sec. II. An approximate model is first derived, and two theoretical procedures, a small aperture procedure and a Babinet equivalence procedure, are explained. The Babinet

equivalence procedure is illustrated by application to a sharp right angle bend in strip line.

The equivalent circuit parameters for a number of specific discontinuity structures are presented in the Appendix. The conductance expression for the radiating slot was derived in previous work by the writer, but all others were obtained by means of the approximation procedures outlined in Sec. II.

In Sec. III, the theoretical results are compared with the available measured data. In this connection, the writer is indebted to Mr. W. E. Fromm, Dr. E. G. Fubini, and Mr. H. Keen of the Airborne Instruments Laboratory for their data on the round hole and the gap in the inner conductor of strip line and for the VSWR of a right angle bend in strip line, and to Dr. A. D. Frost of Tufts College for his data on the VSWR vs. angle of bend for a bend of arbitrary angle in strip line. The measurements at Tufts College were taken with a variable short circuit and the data analyzed by means of Deschamps' method. The measurements on the round hole and the gap, taken at the Airborne Instruments Laboratory, were made using a novel and interesting perturbation scheme due to H. Keen.

The details of the derivations of the theoretical expressions are not given here but will be included in a more comprehensive report issued by the Polytechnic Institute of Brooklyn under a contract with the Air Force Cambridge Research Center.

II. Theoretical Methods

1. The Approximate Model

The balanced strip transmission line considered here, with an inner conductor of zero thickness, is indicated in Fig. 2. Because of the formidable task, mentioned above, of obtaining the appropriate Green's function for the geometry of Fig. 2, the approximate model of Fig. 3 is employed in the determination of the equivalent circuit parameters for discontinuities in the line. The width D of the line is related to the actual center strip width w by

$$D = b K(k)/K(k'), \quad (1a)$$

where

$$k = \tanh(\pi w/2b),$$

and where $K(k)$ is the complete elliptic integral of the first kind of modulus k . When $w/b > 0.5$, corresponding roughly to characteristic impedances less than 100 ohms, D is given to an excellent approxi-

mation by

$$D = w + \frac{2b}{\pi} \ln 2 \quad (1b)$$

The line has electric walls (short circuits) for the top and bottom plates and magnetic walls (open circuits) for the side walls. In effect, then, the approximate model is a portion, of width D , of a parallel plate transmission line of infinite width.

The evolution of this model from the actual geometry is shown in Fig. 4. The fringing of the electric field lines of the dominant mode in the actual geometry, shown in Fig. 4(a), may be compensated for by extending the inner conductor width and placing magnetic walls on the sides of the line. If the extended width has the value D specified by (1a), structures (a) and (b) of Fig. 4 have identical characteristic impedance values. Structure (b) may be viewed as two identical portions of parallel plate transmission line placed back to back, and any incident wave will divide equally into both portions. Since any balanced discontinuity in the line will reflect the same proportion of the incident wave in both top and bottom portions, and since this proportion is also that for the total wave, it is not necessary when dealing with normalized quantities to retain both halves of the line. Thus, in the analysis of balanced discontinuities which are described in terms of normalized quantities one need consider only one-half of the structure and may thus employ the approximate model of Fig. 3.

The equivalent structure of Fig. 4(b) has been applied by the writer to the evaluation of the normalized conductance of a radiating slot cut in one of the outer conductors¹. (Since this discontinuity structure is unbalanced, both halves of Fig. 4(b) were necessary). The result obtained was compared with that derived via a rigorous conformal mapping procedure using the original strip line geometry of Fig. 2, and the agreement was excellent. The results for the radiating slot are included herein as Sec. A-1 of the Appendix. For many discontinuities, the use of the approximate model should yield very satisfactory approximate results.

As an almost direct consequence of the approximate model one can obtain the equivalent circuit parameters of a gap in the inner conductor of strip line (see Fig. 5). Since the gap extends completely across the width of the inner conductor, the discontinuity may be approximated by a gap extending completely across the bottom face, say, of the approximate model, and therefore by a gap of infinite width in a parallel plate waveguide of infinite width. The solution for the latter situa-

tion is obtainable from the already available solution for the E plane slit coupling of rectangular guides, given on pp. 373-375 of the Waveguide Handbook². (It may be noted that parameter B_p of the equivalent circuit of Fig. 7.1-1, on p. 374, should be a capacitance rather than an inductance as given there.) The equivalent circuit parameters for the gap in strip line are presented in Sec. A-3 of the Appendix, and the comparison of theoretical results with measured data is given in Sec. III. The theoretical expressions yield the centerline representation directly; the results for the edge representation are obtained by shifting the centerline representation results through the appropriate lengths of transmission line.

An asymptotic form, valid for $s \gg w$, for the parameter B_A/Y_0 of the edge representation is also given in Sec. A-3. This form is obtained from the results for the E-plane bifurcation presented on pp. 253-255 of the Waveguide Handbook².

2. Small Aperture Procedure

One very useful procedure for obtaining reasonably accurate approximate values for the circuit parameters of a discontinuity structure employs well-known "small aperture" or "stored power" considerations together with a variational expression. An aperture type variational expression, for series type elements, for example, may be written generally as

$$\frac{B}{Y_0} = \frac{\text{Stored Power}}{Y_0 [\text{Voltage}]^2} \quad (2)$$

where a trial aperture electric field E must be inserted into both numerator and denominator. The voltage term is then of the form

$$\text{Voltage} = \iint_{\text{aperture}} \mathbf{n} \times \mathbf{E} \cdot \mathbf{H} ds, \quad (3)$$

where \mathbf{H} is an appropriate mode function which depends on the nature of the excitation of the discontinuity and on the type of waveguide. The determination of the stored power is always the formidable portion of any such evaluation, since it involves integrations over all the higher modes of the waveguide.

The use of the "small apertures" or "stored power" considerations avoids the necessity for a separate evaluation of the numerator of (2). Instead, one seeks the already available solution for a similar problem in a different waveguide cross-section. If the guide walls which are different in the two problems do not substantially influence the stored power (i.e., speaking loosely, the distortion

of the field lines) in the neighborhood of the discontinuity, a property which holds for "small apertures", these considerations state that the stored powers for the two problems may be taken to be equal. The stored power expression is thus abstracted from this similar problem by setting it up also in variational form, even though it may not have been solved originally in this manner. The similar problem must, of course, involve a discontinuity of the same shape and with the same manner of excitation as in the original problem. The desired susceptance value is then equal to the already known susceptance value multiplied by the ratio of the pertinent denominators.

This procedure has been employed in obtaining the equivalent circuit parameters given in Sec. A-2 of the Appendix for the round hole in the inner conductor of strip line. The theoretical results apply directly to the centerline representation; the results for the edge representation are found by shifting the centerline representation results through the appropriate lengths of transmission line. Two different variational expressions are required; one for parameter B_A/Y_0 , obtained by symmetrical electric field excitation of the hole, and one for the parameter combination $(B_A/Y_0 + 2B_p/Y_0)$, resulting from anti-symmetrical electric field excitation. The similar problem, for which the solution is given in the Waveguide Handbook² on pp. 375, 376, is that of E-plane aperture coupling of rectangular waveguides. The details of the derivations will not be presented here.

Other discontinuity structures for which the equivalent circuit parameters may be found in this manner are the small slot in the inner conductor of strip line (shown in Fig. 1(b)), and the vertical post indicated in Fig. 6. This post may be of round, elliptical or rectangular cross-section, and may be perfectly conducting or made of dielectric material. Finally, the parameters for a hole of elliptical, rather than round, shape may also be found by using the Waveguide Handbook pages mentioned above in connection with the round hole.

3. Babinet Equivalence Procedure

When the Babinet equivalent of the approximate model of Fig. 3 is taken, one has available almost immediately the solutions for a variety of discontinuities in strip line. The Babinet equivalent of a structure is found by replacing all magnetic walls by electric walls and vice-versa, and by replacing lines of E by lines of H and lines of H by lines of $-E$.

The equivalent circuit of the original discontinuity structure is then the equivalent circuit dual to that describing the Babinet equivalent discontinuity structure. The numerical values of the corresponding dual elements are identical.

The line which is the Babinet equivalent of the approximate model of Fig. 3 is shown in Fig. 7, and is seen to be another section of parallel plate line rotated through 90° . Its use in the solution of discontinuity problems will be illustrated with reference to the sharp right angle bend in the inner conductor of strip line.

A top view of the inner conductor of a right angle bend in strip line is shown in Fig. 8(a), while a top view of the same bend in the approximate model is indicated in Fig. 8(b). The bend in the Babinet equivalent line is illustrated by Fig. 8(c), and is seen to be a right-angle E-plane bend (or E-plane corner) in parallel plate transmission line. One can find the equivalent circuit for the E-plane corner at the terminal planes T_1 and T_2 in the Babinet equivalent line by looking up pp. 312, 313 of the Waveguide Handbook². This circuit is shown in Fig. 9(a). The equivalent circuit appropriate to the approximate model of Fig. 8(b) at the terminal planes T_1 and T_2 is thus the dual of that of Fig. 9(a), and is given in Fig. 9(b). The specific circuit parameters, which are given in Sec. A-4 of the Appendix, are obtained by making the appropriate dual substitutions in relations 3(b) and 4(b) on p. 313 of the Waveguide Handbook.

The equivalent circuit parameters for the sharp bend of any angle, the sudden change in width and the Tee junction, given in the Appendix, have all been obtained by the use of the Babinet equivalent line. Although not listed in the Appendix, the results for a circular bend, a junction between a straight length and a tapered length of line, and a 120° Y junction either have been or can also be determined by this procedure. The sharp bend of any angle is included here since measured data are available which permit comparison with the theoretical results.

The sudden change in width and the Tee junction are presented here, even though no measurements are available, because of features of interest in their equivalent circuits. For the change in width, one finds a series inductance characterizing the discontinuity, in contrast to the shunt inductance obtained for a change in width in rectangular waveguide and to the shunt capacitance obtained for a change in height in rectangular guide or a change in inner or outer conductor radius in coaxial line. The

Tee Junction equivalent network possesses H plane symmetry, as would be expected, but its reference plane in the stub line is not shifted by $\lambda/4$ as is the case for the corresponding form of the network in an H plane Tee in rectangular waveguide.

In some of these discontinuities some error may be present in the location of the appropriate reference planes. For example, in the change in width, the reference plane T in Sec. A-6 of the Appendix is seen not to coincide with the plane of the actual change in width but is located a distance $(b/\pi)\ln 2$ away from it. This distance is expected to be more accurate if a significant change in width is present; if the change in width is small, however, this distance is expected to be less. Small errors of this type in the location of the reference planes are inherent in the Babinet equivalence procedure.

III. Comparison with Measured Data

With the exception of the radiating slot, which was discussed elsewhere¹, measured data are available for only three of the strip line discontinuity structures listed in the Appendix. These are the round hole, the gap, and the sharp bend.

1. Round Hole in the Inner Conductor (Sec A-2):

The measured data on this discontinuity structure comes from the Airborne Instruments Laboratory, and was obtained using the perturbation scheme of H. Keen. The measurements were taken at a wavelength of approximately 10 cms., on lines of 50 and 75 ohms characteristic impedance. Both the measurements and the theoretical results apply directly to the centerline representation so that the comparison is best made there. The comparison is also a very sensitive one because the parameter values are so small, or so large, depending on the parameter (i.e., the round hole does not exert a very pronounced disturbance on the incident wave). Such a comparison is indicated by the curves of Figs. 10 and 12, for lines of 75 and 50 ohms characteristic impedance, respectively. The theoretical values are indicated by the solid lines, while the measured data are represented by the indicated points and the dashed lines connecting them. It is seen that the agreement is reasonably good. One notes that all of the circuit elements are inductive in this representation.

Comparison between the measured and theoretical values at the edge representation is afforded by Figs. 11 and 13. It is seen that the shunt arms have now become capacitive, and that these parameter values have substantially increased

in magnitude. In addition, the agreement between measurement and theory is now almost perfect over most of the range. The reason for these effects is that the parameter values have now incorporated the length of transmission line between the centerline and edge representations. The more informative representation for this discontinuity is therefore the centerline representation.

The formulas for the equivalent circuit parameters, though simple in form, are more than adequate for engineering purposes.

2. Gap in the Inner Conductor (Sec. A-3):

The measured data for this structure also comes from the Airborne Instruments Laboratory, and was obtained using a perturbation scheme somewhat modified from that employed for the round hole, but also due to H. Keen. Since the gap exerts a much more pronounced effect on the incident wave than does the circular hole, the perturbation procedure is less accurate for the case of the gap, even with the modifications employed. The measured data, which were taken at a wavelength of approximately 10 cms. on a line of 50 ohm characteristic impedance, are still reliable, but are therefore not as trustworthy as those for the round hole.

The theoretical results apply directly to the centerline representation while the measurements were taken at the edge representation. This situation requires a shift of the theoretical values to the edge representation, or a shift of the measured data to the centerline representation, in order to effect a comparison. The comparisons at both of these reference plane locations are presented in Fig. 14. It is seen that there exists a small but consistent discrepancy between the theoretical and measured values for all of the parameters. The possible errors introduced by the use of a perturbation scheme for the measurements may account for part of the discrepancy, but it is still likely that the approximation inherent in the theoretical formulas is the main contributor. At any rate, it is felt that the agreement is sufficiently close for engineering purposes. The line in the edge representation made up of alternate dots and dashes is the asymptotic value of B_A/Y_0 , obtained from the last expression in Sec. A-3 of the Appendix. This latter relation is more reliable than the other theoretical expression in the range $s/w > 1$. It is also in better agreement with the measured data in that range.

3. Sharp bend of any angle Secs. A-4,5:

Complete equivalent circuit data on sharp bends are not available, but the insertion VSWR of a particular right-angle bend has been taken by the Airborne Instruments Laboratory, and the variation of the insertion VSWR with the angle of the bend has been supplied by Tufts College. The theoretical value of the insertion VSWR is readily obtained from the equivalent circuit parameter values by means of the relations in Sec. 8 of the Appendix.

The right-angle bend measured by the Airborne Instruments Laboratory possessed an inner conductor width of 0.625" and an outer plate separation of 0.500", and was measured at a wavelength of 10.0 cms. The measured VSWR value was 2.01; the theoretical value is 2.15. The agreement is considered reasonably good.

The bend of arbitrary angle measured by Tufts College was located in polystyrene filled line, with an outer plate separation of 0.500" and an inner conductor width of 0.400", and was measured at a frequency of 2800 mc. A comparison between the measured and theoretical values of insertion VSWR as a function of the angle of the bend is presented in Fig. 15. The theoretical values are represented by the solid line, the measured values by the indicated points. The agreement is seen to be good except for the region of angles below 40° or so. In this region the theoretical values vary in monotonic fashion while the measured values follow the dashed curve. The measured data were obtained by the use of a variable short circuit and analyzed by Deschamps' method, a procedure which should yield a high degree of precision. However, since there seems to be no theoretical justification for a minor peak in the region of low angles, it is possible that this behavior may be due to certain mechanical irregularities or defects. At any rate, this question will be looked into further by Tufts College.³

Since the VSWR values are independent of the specification of reference planes, the above comparison with measured data sheds no light on the accuracy of the reference plane locations. Certain questions concerning this accuracy have been raised in connection with the discussion in Sec. II on the Babinet equivalence procedure.

References

1. A. A. Oliner, "The radiation conductance of a series slot in strip transmission line", Paper No. 43.3, IRE

National Convention, March 1954, New York City. Also, this material forms part of a forthcoming Hughes Aircraft Company report by the writer entitled "Theoretical Studies in Strip Transmission Line".

2. N. Marcuvitz, "Waveguide Handbook", Vol. 10, Radiation Laboratory Series, McGraw-Hill Book Co., New York, 1951.
3. Private communication from Dr. A. D. Frost.

Acknowledgment

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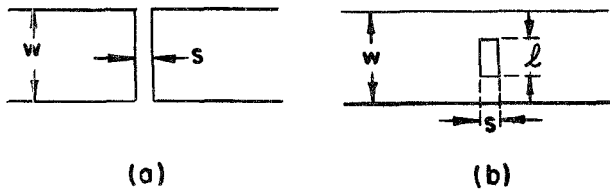


Fig. 1 - Development of a series resonant slot.

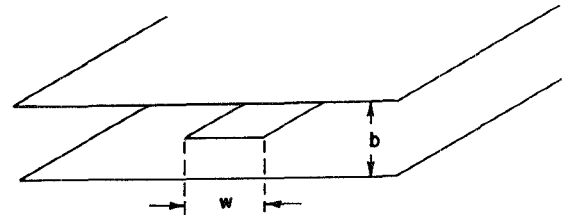


Fig. 2 - The balanced strip transmission line.

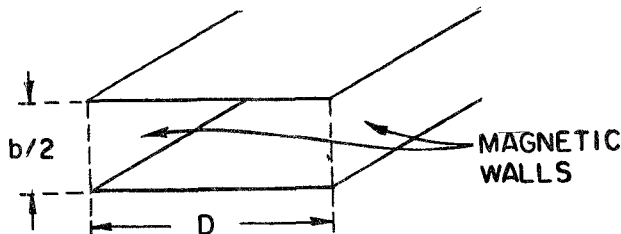


Fig. 3 - The approximate model.

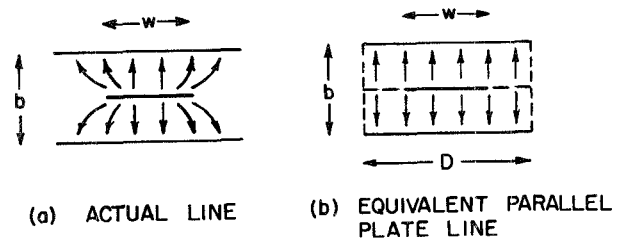


Fig. 4 - Evolution of the approximate model.

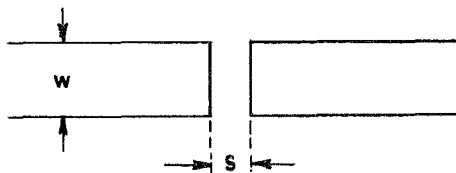


Fig. 5 - Gap in inner conductor of strip line.

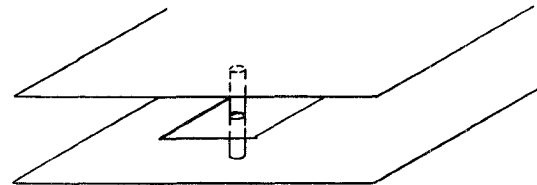


Fig. 6 - Vertical post in strip line.

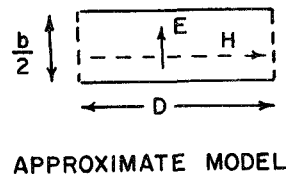
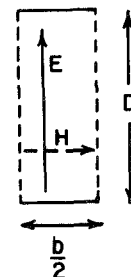


Fig. 7 - The Babinet equivalent line.



BABINET EQUIVALENT LINE

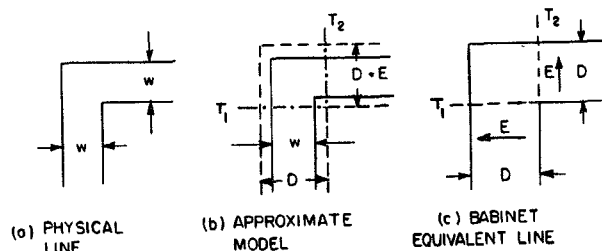


Fig. 8 - Right angle bend.

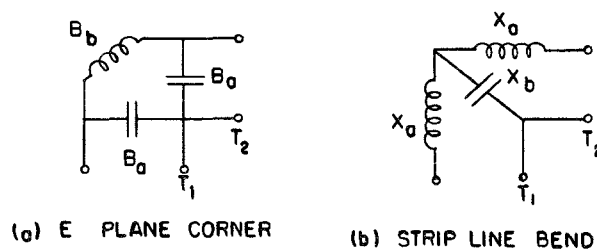


Fig. 9 - Dual equivalent circuits.

ROUND HOLE IN STRIP LINE

75 ohm line

CENTERLINE REPRESENTATION

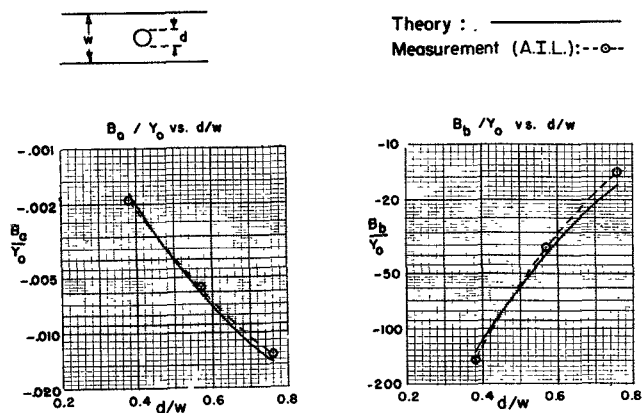


Fig. 10

ROUND HOLE IN STRIP LINE

75 ohm line

EDGE REPRESENTATION

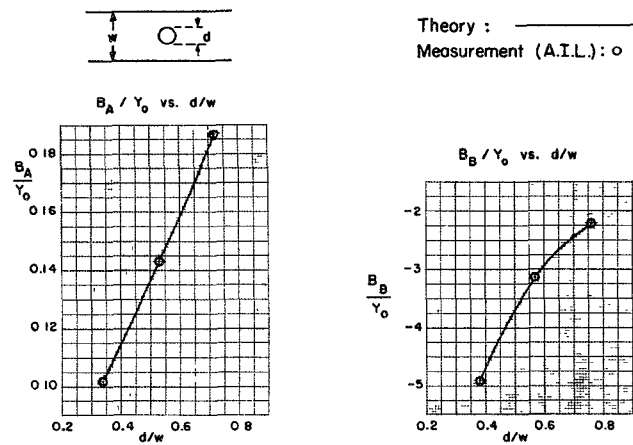


Fig. 11

ROUND HOLE IN STRIP LINE

50 ohm line

CENTERLINE REPRESENTATION

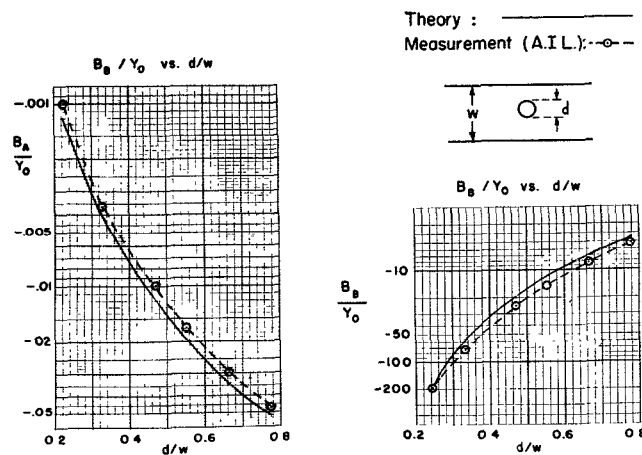


Fig. 12

ROUND HOLE IN STRIP LINE

50 ohm line

EDGE REPRESENTATION

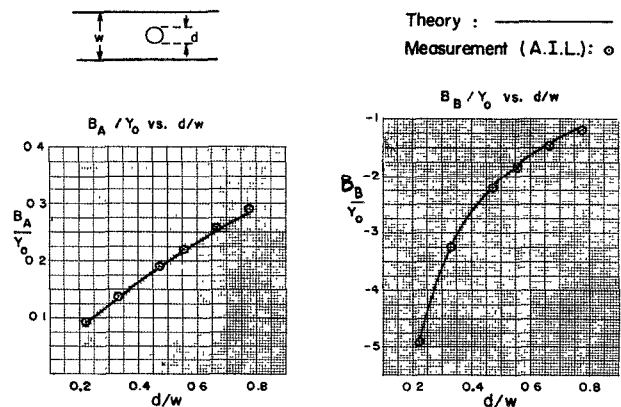
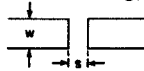


Fig. 13

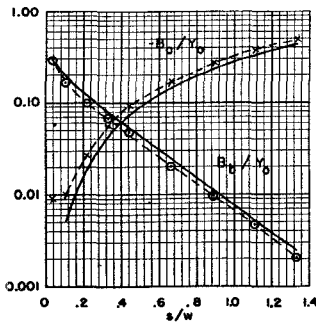
GAP IN STRIP LINE



Theory : —————

Measurement (A.I.L.) : — x —
— o —

CENTERLINE REPRESENTATION
 B_o/Y_o and B_b/Y_o vs. s/w



EDGE REPRESENTATION
 B_A/Y_o and B_B/Y_o vs. s/w
Asymptotic theoretical value
of B_A/Y_o : ————

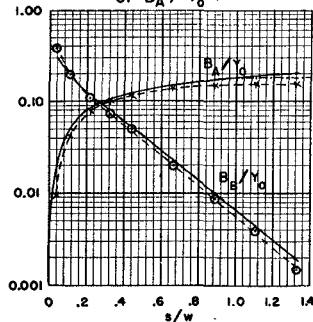


Fig. 14

SHARP BENDS IN STRIP LINE

VSWR of 90° bend

measurement (A.I.L.): 2.01

theory: 2.15

VSWR of bend vs. angle θ

measurement (Tufts) : — o —
theory : ————

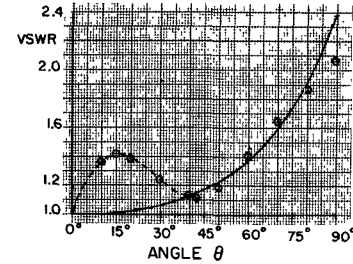
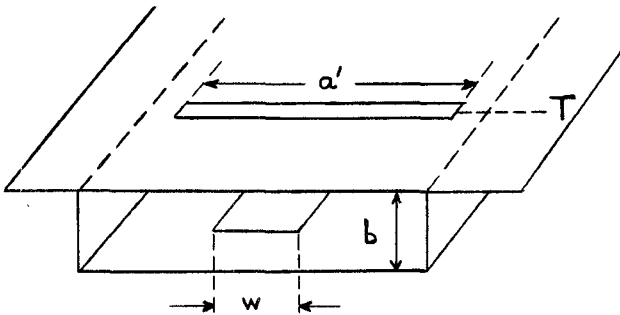


Fig. 15

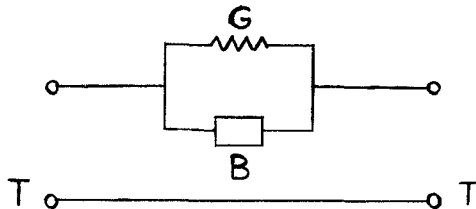
APPENDIX: RESULTS FOR SPECIFIC DISCONTUITY STRUCTURES

A-1. CONDUCTANCE OF RADIATING SLOT

(a) Physical structure:



(b) Equivalent circuit:



(c) Conductance parameter:

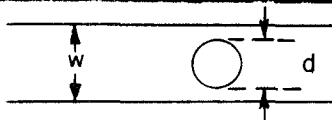
$$\frac{G}{Y_o} = \frac{16}{3\pi} \frac{b}{D} \left(\frac{a'}{\lambda}\right)^2 \left[1 - .374 \left(\frac{a'}{\lambda}\right)^2 + .130 \left(\frac{a'}{\lambda}\right)^4 \right]$$

where D is given by Eq. (1a) or (1b).

At resonance, $\frac{B}{Z_o} \approx \frac{200}{Z_o}$.

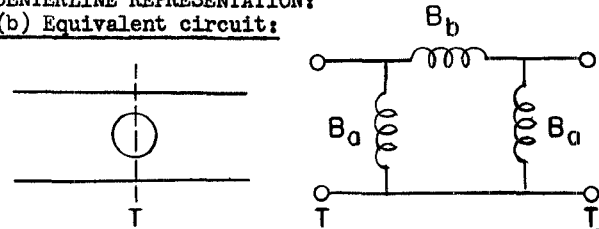
A-2. ROUND HOLE IN INNER CONDUCTOR

(a) Physical structure: (top view of inner conductor)



CENTERLINE REPRESENTATION:

(b) Equivalent circuit:



(c) Circuit parameters:

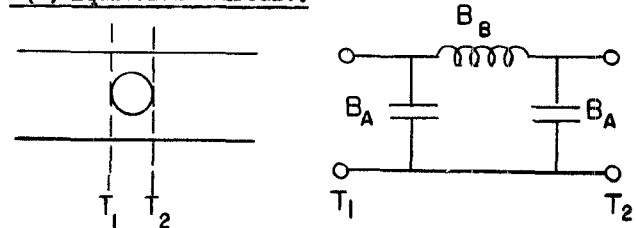
$$\frac{B_b}{Y_o} = - \frac{3}{4\pi} \frac{\lambda b D}{d^3},$$

where D is given by Eq. (1a) or (1b).

$$\frac{B_a}{Y_o} = \frac{1}{4 (B_b/Y_o)}$$

EDGE REPRESENTATION:

(b) Equivalent circuit:



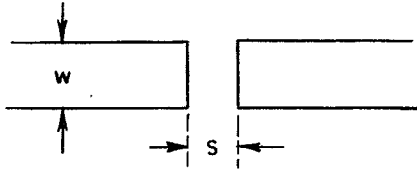
(c) Circuit parameters:

$$\frac{B_A}{Y_o} = \frac{1 + (B_b/Y_o) \cot(\pi d/\lambda)}{\cot(\pi d/\lambda) - (B_b/Y_o)}$$

$$\frac{B_B}{Y_o} = \frac{1}{2} \left[\frac{1 + 2 (B_b/Y_o) \cot(\pi d/\lambda)}{\cot(\pi d/\lambda) - 2 (B_b/Y_o)} \right] - \frac{1}{2} \frac{B_A}{Y_o}$$

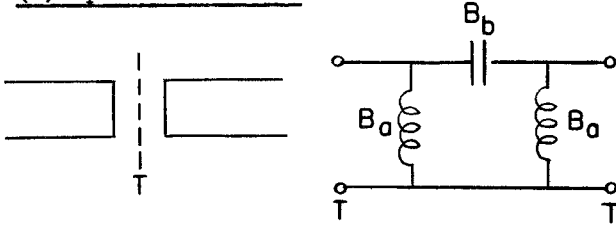
A-3. GAP IN INNER CONDUCTOR

(a) Physical structure: (top view of inner conductor)



CENTERLINE REPRESENTATION:

(b) Equivalent circuit:



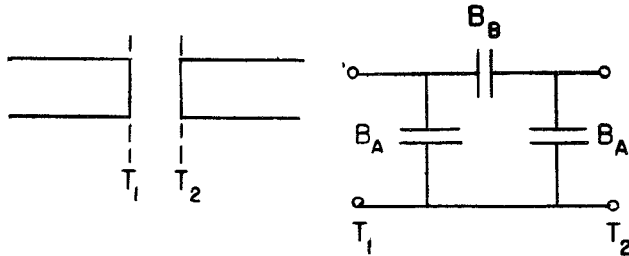
(c) Circuit parameters:

$$\frac{B_a}{Y_o} = -\frac{2b}{\lambda} \ln \cosh \frac{\pi s}{2b}$$

$$\frac{B_b}{Y_o} = \frac{b}{\lambda} \ln \coth \frac{\pi s}{2b}$$

EDGE REPRESENTATION:

(b) Equivalent circuit:



(c) Circuit parameters:

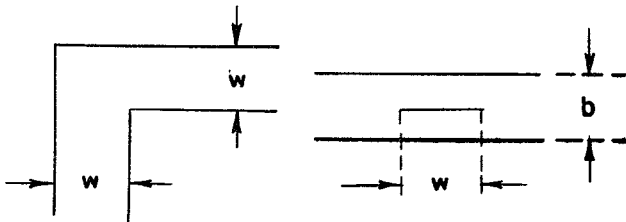
$$\frac{B_A}{Y_o} = \frac{1 + (B_B/Y_o) \cot(\pi s/\lambda)}{\cot(\pi s/\lambda) - (B_B/Y_o)}$$

$$\frac{B_B}{Y_o} = \frac{1}{2} \left[\frac{1 + (2B_B/Y_o + B_A/Y_o) \cot(\pi s/\lambda)}{\cot(\pi s/\lambda) - (2B_B/Y_o + B_A/Y_o)} \right] - \frac{1}{2} \frac{B_A}{Y_o}$$

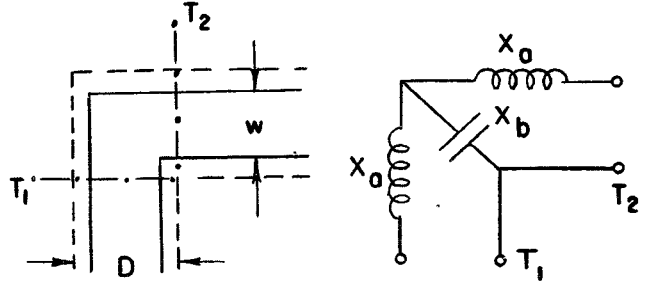
$$\text{For } s \gg w: \frac{B_A}{Y_o} \sim \tan\left(\frac{2b}{\lambda} \ln 2\right)$$

A-4. SHARP RIGHT ANGLE BEND

(a) Physical structure:



(b) Equivalent circuit:



(c) Circuit parameters:

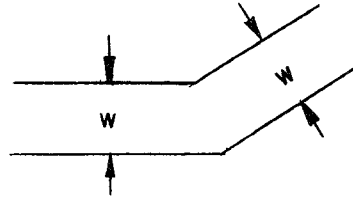
$$\frac{X_a}{Z_o} = \frac{2D}{\lambda} \left(0.878 + 2 \left(\frac{D}{\lambda} \right)^2 \right)$$

$$\frac{X_b}{Z_o} = -\frac{\lambda}{2\pi D} \left(1 - 0.114 \left(\frac{2D}{\lambda} \right)^2 \right)$$

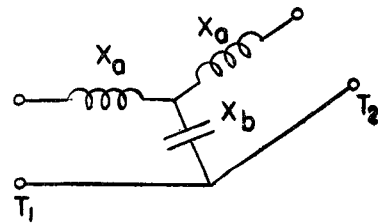
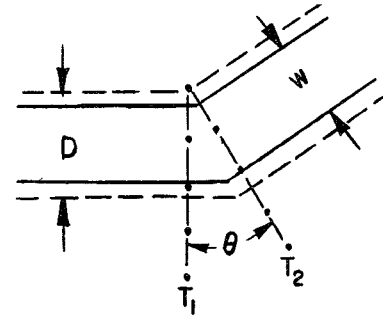
where D is given by Eq. (1a) or (1b).

A-5. SHARP BEND OF ANY ANGLE

(a) Physical structure: (top view of inner conductor)



(b) Equivalent circuit:



(c) Circuit parameters:

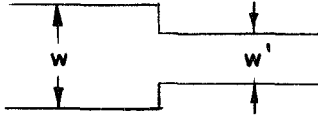
$$\frac{X_b}{Z_o} = -\frac{\lambda}{2\pi D} \cot \frac{\theta}{2}$$

$$\frac{X_a}{Z_o} = \frac{2D}{\lambda} \left\{ \Psi \left[-\frac{1}{2} \left(1 - \frac{\theta}{\pi} \right) \right] - \Psi \left[-\frac{1}{2} \right] \right\}$$

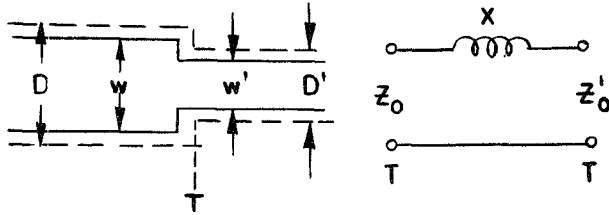
where $\Psi(x)$ is the logarithmic derivative of $x!$, and where D is given by Eq. (1a) or (1b).

A-6. SUDDEN CHANGE IN WIDTH

(a) Physical structure: (top view of inner conductor)



(b) Equivalent circuit:



(c) Circuit parameters:

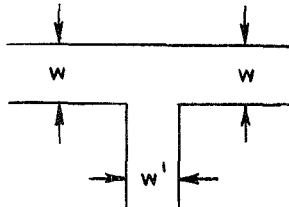
$$\frac{X}{Z_o} = \frac{2D}{\lambda} \ln \csc \frac{\pi D'}{2D}$$

where D is given by Eq. (1a) or (1b).

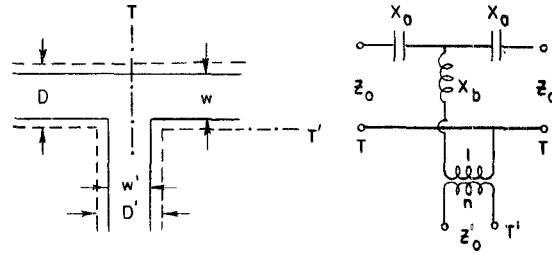
$$\frac{Z'_o}{Z_o} = \frac{D}{D'}$$

A-7. TEE JUNCTION

(a) Physical structure: (top view of inner conductor)



(b) Equivalent circuit:



(c) Circuit parameters:

$$\frac{Z'_o}{Z_o} = \frac{D}{D'}$$

$$n = \frac{\sin(\pi D'/\lambda)}{\pi D'/\lambda}$$

$$\frac{X_a}{Z_o} = -\frac{D'}{\lambda} \frac{D'}{D} \frac{\pi^2}{16} n^2$$

$$\frac{X_b}{Z_o} = \frac{1}{n^2} \frac{2D}{\lambda} \left[\ln \csc \frac{\pi D'}{2D} + \ln 2 + \frac{\pi D'}{6D} + \frac{3}{2} \left(\frac{D'}{\lambda} \right)^2 \right]$$

where D is given by Eq. (1a) or (1b).

A-8. INSERTION VSWR OF FOUR-TERMINAL STRUCTURE

If the structure is characterized by a Tee network, with parameters X_a and X_b , then one can find the normalized susceptance of the pure shunt network equivalent (at shifted reference planes) from:

$$B \equiv \frac{B}{Y_o} = -\frac{1}{(X_b/Z_o)} \left[1 + \frac{X_a}{Z_o} \left(\frac{X_a}{Z_o} + 2 \frac{X_b}{Z_o} \right) \right]$$

The insertion VSWR is then given by:

$$VSWR = \frac{\sqrt{4 + B^2} + B}{\sqrt{4 + B^2} - B}$$

For a VSWR calculation reference planes need not be specified.